**Theory and practice in mathematics teacher education**

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**Resumen.** Se examina el desafío de establecer la interacción entre teoría y práctica en la formación del profesor de matemáticas utilizando la teoría antropológica de lo didáctico (TAD). Se describe el problema teoría-práctica tanto en un contexto internacional como danés. Después de una breve introducción a la TAD, se ilustra cómo pueden ser utilizadas las nociones de transposición didáctica y praxeología para analizar la relación teoría-práctica con un ejemplo sobre adición de fracciones. Sobre este análisis se combinan los dos modelos en un modelo más comprehensivo para describir y analizar la formación del profesor de matemáticas centrada en el problema teoría-práctica.

**Résumé.** Le défi d’établir l’interaction entre la théorie et la pratique dans la formation des enseignants des mathématiques est examiné par l’utilisation de la théorie anthropologique du didactique (TAD). Le problème de l’articulation théorie-pratique est décrit à la fois dans un contexte international et dans le contexte danois. Après une brève introduction à la TAD, le problème est illustré par un exemple portant sur l’addition de fractions pour montrer comment les notions de transposition didactique et de praxéologie peuvent être utilisées pour analyser la relation entre théorie et pratique dans cette situation. En nous appuyant sur cette analyse, les deux modèles sont combinés pour créer un modèle plus complet en vue de décrire et d’analyser la formation des enseignants des mathématiques en vue du problème de la relation entre théorie et pratique.

**Abstract.** The challenge of establishing interplay between theory and practice in mathematics teacher education is examined by use of the anthropologic theory of the didactic (ATD). The theory-practice problem is described both in an international and a Danish context. After a brief introduction to ATD, it is illustrated with an example on addition of fractions how the notions didactic transposition and praxeology can be used to analyze the theory-practice relation in this situation. Building on this analysis the two models are combined into a more comprehensive model for describing and analyzing mathematical teacher education focusing on the theory-practice problem.
1. Introduction

Establishing coherence between theory and practice is one of the main challenges in mathematics teacher education. In Denmark more than four out of ten student teachers experience a lack of coherence between the teaching of general educational science taking place at the university college and teaching practice in schools (Jensen et al., 2008). Throughout the last decades teacher education has become increasingly academic – which can be seen as positive – but concurrently, the practice situation at diverse schools has become much more challenging, particularly because of an increasing social and ethnic segregation that affect schools in disadvantaged neighborhoods. Therefore, many student teachers want practical teaching tools and not academic theories and this of course entails a risk of widening the gap.

The lack of coherence between theory and practice in teacher education is not only a problem in Denmark. It occurs in different ways but it is described and researched internationally. Bergsten and Grevholm (2005) point out two different didactical divides. The first divide is between the theoretical knowledge learned in the study of mathematics or pedagogics, respectively, and the practice of mathematics teachers in school. The conditions for the divide between theoretical knowledge in pedagogy and teaching practice will not be treated in this paper – the focus is on mathematics teaching. Mathematics, which is an academic subject mainly developed through research at the universities (but also in e.g. commercial and technical contexts), must be adapted to school mathematics suited for the particular educational context and aim. Student teachers will, of course, meet with both scholar and school mathematics in teacher education, but it is absolutely essential for them to learn to transform from one to the other, e.g. when they have to derive the mathematical points from teaching materials or elaborate concrete teaching from a national curriculum.

The second divide is between pedagogical and mathematical knowledge. Academic theories on pedagogy and learning are introduced in the general and common part of teacher education and in mathematics in particular, didactical theories on the teaching and learning of
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mathematics are introduced. Student teachers are supposed to combine their knowledge from the two diverse disciplines in common teacher knowledge usable for teaching practice where problems occur in varied, complex forms. Bergsten and Grevholm (ibid.) point out the lack of theory to unify these two kinds of knowledge in a common pedagogical content knowledge about teaching and learning mathematics in school.

2. Teacher knowledge

Teacher knowledge – and ensuing practices of mathematics teachers – is a very complex matter. According to Schulman (1987) teacher knowledge is comprised of three separate strands:

- **Subject matter knowledge** (SMK). In this case mathematical knowledge.
- **Pedagogical knowledge** (PK).
- **Pedagogical content knowledge** (PCK).

At the universities the three strands are researched and taught separately and rarely combined. One of the main problems in mathematics teacher education is to establish interplay between these three different strands and concrete experiences with teacher knowledge gained from teaching practice. Preparatory lessons at the teachers college and teaching practice are often mentioned as one of the best opportunities to build a bridge between theory and practice but research is inconclusive on this matter (Bergsten et al., 2009, p. 60). Lack of theoretical focus often implies that student teachers experience teaching practice as complex and stressful and therefore fall into short-lived performance without coherence to their learning outcome from the theoretical education. The aim of this paper is to design a theoretical model to examine and analyze what is theoretically required to link theory and practice and how the education at university college, preparatory lessons and teaching practice can be organised to establish circumstances to create this link.

In Denmark, mathematics teacher education for primary and lower secondary school is located in university colleges. University colleges are evolved from the non-academic seminar-tradition and now in many ways
evolving toward more academic standards although they still are not universities.

As in most other countries, teacher education in Denmark consists essentially of three key strands; mathematics, pedagogy and teaching practice. We have a joint occurrence of the three strands – this is what Tatro et al. (2009, p. 18) call concurrent preparation. Although, concurrent preparation – compared to consecutive preparation with three independent periods (ibid., p. 18) – is supposed to better enable the students to correlate theory and practice, many student teachers in Denmark, as previously mentioned, still experience a lack of coherence between the general education located at university colleges and teaching practice.

The divide between pedagogical and mathematical knowledge is further intensified in Denmark by the fact that teacher education and the general routine at schools have developed in disparate ways on this point during the last 20 years. On the one hand, change of laws within teacher education has changed focus from educating the “comprehensive teacher” who teaches almost all subjects – and of whom some teach mathematics – to a more specialized teacher education where each teacher teach primarily two or three “main subjects”. But while teacher education has emphasized to educate mathematics teachers, schools on the other hand still emphasize that pupils must be taught by the fewest possible teachers. Therefore teachers are encouraged to teach more subjects: teacher first, then (e.g.) mathematics teacher. The principle of the comprehensive school where teachers teach more subjects and pupils meet only a few teachers is very strong in Denmark and this, among other things, results in the fact that 37 % of all mathematics teachers are not educated with mathematics as their main subject (Hune, 2009). Student teachers therefore encounter with different views and priorities of the relation between pedagogy and mathematics in teacher education at university colleges and schools. This may complicate the divide and makes it even more difficult to combine practice at school and theory at the university college.

The theory-practice divide can be regarded from two points of view: Firstly, one can consider the problem from theory to practice: Is it
possible – and desirable – to use teaching practice to motivate or even support teacher students’ acquisition of theoretical knowledge? And how do we create a shared frame of reference from teaching practice to interpret the theory? Subject matter knowledge, pedagogical knowledge and pedagogical content knowledge are taught separately at university colleges but are in reality inextricably entwined with each other. The challenge is how to create interplay between the academic theories of mathematics and pedagogy and teaching practice in teacher education. It is crucial to create this interplay e.g. to legitimize the theoretical education and to place school knowledge in a wider context. Secondly, one can consider the problem from practice to theory. In teacher education, the teaching practice must be discussed in the theoretical education to ensure that student teachers’ learning in and from teaching practice is connected to the theoretical education and brings about a critical view on the theories and research from a practical point of view. This requires a common model to describe and analyze mathematical knowledge, teaching and learning at different levels and an ongoing development of teaching practice models to develop mathematical teaching. Inevitably, teachers learn through everyday teaching but their experiences are rarely shared with colleges or other professionals and the knowledge therefore remains individual (Skott, 2001). Lack of explicit knowledge about teacher practice complicates the transition from student teacher to in-service teacher (Winsløw, 2009) and makes the profession vulnerable to criticism (Stiegler & Hiebert, 2009). The problem is how to build up knowledge about how teacher knowledge and teaching practice can be developed in a Danish context. The basic idea of my project is that teacher students should acquire teacher knowledge simultaneously from teaching practice and the theoretical education at the university college. It is of crucial importance for teachers in mathematics to develop their teacher knowledge through a lifelong career and therefore teacher education must offer tools to learn in and from teaching practice.

The next step of my project is to develop institutional possibilities for co-operation between teachers, especially at particular schools, to make sure that the emerged knowledge will be shared and not remains individual. In order to reach this goal it is necessary to design and
establish learning communities that enable student teachers and teachers to try out, discuss and develop theories about teaching and learning in practice. Teachers must learn to develop teaching through a lifelong career and this starts during teacher education. According to Stigler and Hiebert:

“Teacher learning is the key to improve teaching. But not any kind of teacher learning will do. (…) Schools must become places where teachers, not just students, learn”. (Stigler & Hiebert, 2009, pp. 36-37).

One aim of my project is to create different cooperation communities (Jaworski, 2003) where student teachers, in-service teachers and, if possible, teacher education teachers build up knowledge based on mutual experiences with the same mathematical teaching situation, explicitly and systematically planned and evaluated based on a common theoretical framework.

It is a current research topic to identify learning conditions for teachers in a common practice. The aim of my project is to contribute to this topic by researching and developing tools, models and strategies to support student teachers’ learning in and from practice in Danish teacher education across mathematical themes, learner age and school cultures. The purpose of this paper is to design a model to correlate theory and practice in mathematical teacher education on the basis of the anthropologic theory of the didactic (ATD). The model will be used as a theoretical tool to link theory and practice in mathematical teacher education and thus to develop student teachers’ learning in and from teaching practice. Firstly, ATD will be briefly presented in the next section.

3. The anthropologic theory of the didactic

Teaching practice is often described by the didactic triangle (figure 1) consisting of three interrelated elements; teacher, student and content.
However, the three elements are not dealt with to the same extent in didactics research. Historically speaking, didactics research at the European continent, particularly in Germany, has focused on the content whereas research in England and United States has focused mainly on the student and learning theories – the relation between students and content. During the last 15-20 years focus on the teacher has increased in the didactics research, though (Winsløw, 2011b). This paper will focus on the development of mathematics teachers’ didactical and mathematical knowledge.

3.1. Didactic transposition

The complex coherence between practical and theoretical knowledge about teaching mathematics is modeled in the anthropological theory of the didactic as mathematical and didactic praxeologies (Chevallard, 1999). The founder of the theory, Yves Chevallard, has, together with other international researchers, developed a ground-braking research program about the coherence between theory and practice in teaching and the relationship between didactical and mathematical knowledge. The theory is anthropological on a scientific theoretical base as it describes both didactical and content knowledge as founded on, and regarding, concrete human activity. It provides tools to analyze, model and design coherent “units” of human knowledge and practice on an empirical foundation. This makes the theory appropriate to consider what I have pointed out as two main problems in Danish mathematical teacher education – the lack of coherence between theory and practice and between didactical and content knowledge. Furthermore, the theory will be used to analyze and expound how teaching practice can contribute to
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develop teachers teaching knowledge and prepare teacher students for life-long learning in and from practice.

Chevallard names the adaptation from scholar mathematics developed mainly at universities to school mathematics as it occurs in schools the didactic transposition (figure 2). The didactic transposition is divided into three steps. First an external didactic transposition taking place outside the school, by, what Chevallard terms, the noosphere, from scholar mathematics to mathematics meant to be taught as it is described e.g. in curricula. Second an internal didactic transposition taking place inside the school, carried out by the teacher and very often based on a course book system, from knowledge meant to be taught to knowledge actually taught. Very few teachers are involved in the external didactic transposition but internal didactic transposition is everyday work for all teachers. The third step is about what happens in the learning situation, often in the classroom, between the teacher and the pupil – influenced by classmates and the learning environment.

Figure 2. Didactic transposition

Didactic transposition in mathematics teacher education can be illustrated in a model similar to figure 2 but teacher education is characterized by the fact that student teachers are supposed to learn mathematics and to learn to teach mathematics. This means that student teachers have to deal with a meta-didactic transposition: The didactic transposition from scholar mathematics to knowledge actually learnt in teacher education can be provided as a practical example for the teacher students of how to work on the didactic transposition from scholar mathematics to knowledge actually learnt in school. Student teachers
must gain experiences and reflect on their own learning process to acquire tools to accomplish the didactic transposition in their own work as teachers in schools.

It is crucial for student teachers to gain an insight into the didactic transposition by analyzing the changes of knowledge and practice in mathematical activities in different contexts. First of all because it creates a link to bridge the didactical divide between scholar mathematics and school mathematics and secondly because it is essential for teachers to adapt different “kinds” of mathematics to other kinds in their everyday work, for instance to derive mathematical points from teaching materials and place them in a broader theoretical context or to adapt mathematical concepts from the curriculum to relevant mathematical activities for pupils at school. In addition to this, the way the mathematical knowledge is treated in school including the choice of working methods is rooted in scholar mathematics in a wide sense.

Mathematical content can be analyzed by stating a reference epistemological model (REM) (Bosch & Gascón, 2006). A REM describes the mathematical content by posing a multitude of fundamental questions from the different perspectives in the didactic transposition. For instance, rational numbers are defined differently in scholar mathematics and school mathematics. A REM about rational numbers should encompass analysis and answers from the different perspectives to questions such as: What are rational numbers? How to describe them in terms of praxeologies (see page 10-11)? What is their connection to other praxeologies, for instance other number sets? How does one calculate with rational numbers? Etc. The aim of stating a REM is to provide a specific way of looking at the mathematical content, in this particular case rational numbers, to question how mathematics is taught and to make proposals for doing things differently. Having analyzed the answers to questions like these results in a number of didactical decisions about what content to choose at a given level, why and how to teach it but also choices about what to leave out.
3.2. Scale of levels of determination

A fundamental issue in mathematics teacher education is the selection of mathematical contents for teaching pupils at school – “knowledge meant to be taught”. The “classical paradigm of visiting monuments of knowledge” (Chevallard, 2012, p. 3) where content is “sanctified by tradition” (ibid., p. 3) is questioned by pupils, parents, society and researchers and therefore teachers have to justify their choices of content. At first, student teachers are supposed to read the curriculum but professional teacher knowledge includes a deeper understanding of why exact content is chosen. This choice can be analyzed employing didactic transposition. At first glance it might look like a problem determined by conditions and restrictions “immediately identifiable in the classroom; teacher’s and students’ knowledge, didactical material available, software, temporal organization, etc.”, (Bosch and Gascón, 2006, p. 61) but one of the main points in didactical transposition theory is that selection of mathematical content is determined by institutional factors (ibid., p. 56, and Winsløw, 2011a, p. 122) which includes both conditions and restrictions inside and outside the classroom (Bosch and Gascón, 2006, p. 61). This calls for an analysis of the influence from external factors on the selection of mathematical content e.g. school traditions and societal norms. These conditions and restrictions can be analyzed and researched by applying Chevallard’s “scale of levels of determination” (ibid., p. 61) to analyze how a given mathematical content is selected. This part of Chevallard’s theory will not be elaborated in this paper but can be further studied in (Bosch and Gascón, 2006).

3.3. Praxeologies

Chevallard introduces the concept praxeology to model and analyze human activities. Praxeology is a linguistic compound of practice and knowledge (from Greek logos). Human activity is an interrelated combination of practice and knowledge; every human activity (practice) is motivated by thinking and reasoning (knowledge) and practice then again affects knowledge.

A praxeology (figure 3) is described by a 4T-model – a four-tuple \((T,\tau,\theta,\Theta)\) consisting of a Type of tasks \((T)\), a Technique \((\tau)\), a Technology
(θ) and a Theory (Θ). First the four elements will be described briefly and then in the next section the description will be elaborated by providing an example of how it can be used as a theoretical tool to analyze both theory and practice in teacher education.

<table>
<thead>
<tr>
<th>Theoretical block</th>
<th>Practical block</th>
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<tbody>
<tr>
<td>Theory</td>
<td>Technique</td>
</tr>
<tr>
<td>Technology</td>
<td>Type of task</td>
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Figure 3. Praxeology

The task is what initiates human activity – in school the task is supposed to motivate the pupils to perform actions that will give them the opportunity to learn. To solve a task one has to use a technique – in mathematics this could for instance be rules of arithmetic. A technique is often useable for a type of tasks. Type of tasks and technique combined is called the practical block – concerning the know-how. Technology is made of explanations and arguments to back up the technique – in mathematics it can be theorems, proofs and properties. Theory is made of explanations and arguments to back up technology – in mathematics it can be definitions, axioms etc. Technology and theory constitute the theoretical block – concerning know that and why.

3.4. Mathematical organizations and didactic organizations

Praxeologies can, as illustrated above, have references to mathematical tasks and situations – or content knowledge in Schulman’s (Schulman, 1987) terms. These are named mathematical organizations. In many cases, a type of tasks can be solved by using the same technique – these are named point organizations. Similarly, praxeologies unified by technology are termed local organizations and praxeologies unified by theory are termed regional organizations.
Praxeologies can also have references to how to learn and teach subjects e.g. mathematics – or pedagogical knowledge in Schulman’s (ibid.) terms. These are called didactic praxeologies. Didactic praxeologies concerning pupils or students learning are termed pupils’ or students’ didactic praxeologies and didactic praxeologies concerning teaching a mathematical praxeology are named teacher’s didactic praxeologies (Barbé et al. 2005). Mathematical and didactic organizations are interdependent – didactic praxeologies are meaningless without mathematical (or other subjects) praxeologies – with Chevallard’s words the two are co-determined.

Teacher’s didactic praxeologies are utilized to describe and analyze what to do (type of tasks) when teaching a mathematical organization, how to do it (technique) and why to do it this way (theory block). Explanations of the techniques (technology) are often interpreted from the context and accepted as such (Madsen & Winsløw, 2009). Consequently, technology and theory are often linked and we end up with a 3T-model: Type of task, Technique and Theory block.

3.5. Moment of didactic processes

Didactic and mathematical praxeologies are correlated through didactic processes concerning what Shulman (1987) terms pedagogical content knowledge. In ATD the didactic process of students creating or re-creating a mathematical organization \((T_\iota, \tau_\iota, \theta, \Theta)\) is described by six didactic moments (Barbé et al., 2005):

1. The moment of the first encounter
2. The exploratory moment
3. The technological–theoretical moment
4. The moment of the technical work
5. The institutionalization moment
6. The evaluation moment

These six moments are crucial moments or dimensions in the learning process and are linked to the praxeological structure given to mathematical contents. The first moment is the student’s first encounter with the mathematical organization often represented by a first type of tasks \(T_i\). After this, in the exploratory moment, the student explores
similar tasks $T_i$ to elaborate a suitable technique $\tau_i$ to solve the group of similar tasks. In the technological-theoretical moment the techniques are explained and justified and the theory block $(\theta,\Theta)$ is constituted. The technical moment consist in working on the technique $\tau_i$ to improve it and to develop students’ skills and routines. The last two moments are closely connected. In the institutionalization moment the mathematical organization is related to other mathematical organizations through the theory blocks $(\theta,\Theta)$. At last, at the evaluation moment the type of tasks $T_i$ is related to the developed mathematical organization and the process is validated.

Each of the six moments contains didactic tasks which can be taken as the basis of didactic praxeologies. In teacher education the didactic moments can be used as a tool to combine mathematical and didactic praxeologies and show how the two are inextricably intertwined. Again, the moments can be identified and analyzed in the theoretical teaching of mathematics in teacher education and applied by the students in their own teaching in school.

4. Addition of fractions - an example

In the next section addition of fractions will be described in terms of praxeologies to show how the 4T-model can be applied to describe and analyze this topic in mathematics teacher education, in order to show how the model can create correlation between theory and practice. It is a general point that the theory of praxeologies is used explicitly in the theoretical part of the education at university college to describe and analyze the scholar mathematics and knowledge meant to be taught, to ensure that the student teachers are familiar with the theory and thus capable of using it to plan, observe and analyze their own teaching practice. After the description the example is used to identify crucial issues and connections between different parts of mathematical teacher education and, on this basis, design a model. The aim of the model is to help identify and analyze different types of theory-practice problems in mathematical teacher education and thus propose ways to overcome them.
The example is hypothetical and has not been carried out yet, but it will be at a later time. The starting point of the course plan is the question: What is the sum of $\frac{a}{b}$ and $\frac{c}{d}$ – e.g. what is the sum of $\frac{1}{6}$ and $\frac{3}{4}$?

The usual techniques, as all the students of course know very well, is to find a common denominator including the lowest common multiple, to extend fractions, to add fractions with a common denominator and to reduce fractions. To add $\frac{1}{6}$ and $\frac{3}{4}$ is, of course, an easy task for student teachers whereas adding $\frac{a}{b}$ and $\frac{c}{d}$ is challenging for most student teachers even though some would say that this is knowledge they should have learned already in high school. Because the shift in level of abstraction from a concrete example to the general rule is challenging for most student teachers it is a suitable starting point for studying the technology – explanations of rules of arithmetic of rational numbers including e.g. if $\frac{a}{b}$ is an arbitrary fraction and $n$ an arbitrary integer, not 0, then $\frac{a}{b} = \frac{n \cdot a}{n \cdot b}$. Theory is framing and justifying technology in this case for instance framing the definition of rational numbers, $\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{N} \right\}$.

The starting analysis of addition of fractions includes elements from both school and scholar mathematics. After this, the topic can be treated from different perspectives. I choose knowledge meant to be taught as my next point of view. The aim of teaching fractions after 9th grade is described in the Danish curriculum, termed Common Goals 2009:

- working with numbers and algebra – calculating with fractions e.g. in connection with solving equations and algebraic problems.

(Common Goals 2009, p. 9, translation by the author).

And later in a paragraph about how to achieve the aims:

The pupils still develop methods of calculation to increase their number comprehension. In this course plan focus is on developing methods on arithmetic with fractions, calculation of percentage and solving equations. The work includes mental arithmetic, arithmetic with written notes and use of calculator.
Fractions are applied in contexts that occur naturally. Arithmetic of fractions is adapted considering the use when solving equations and other algebraic themes. (Common Goals, p. 26, translation by the author).

It appears from the quotation from Common Goals 2009 that aims and terms about how to achieve the aims are formulated in very open phrases. Pupils must learn how to calculate with fractions but the curriculum does not frame in details to what extent. The example of a task chosen above, \( \frac{1}{6} + \frac{3}{4} \), will probably be considered included in the latter quotation above (Common Goals 2009, p. 26) and similar examples can be found in every text book for lower secondary school and exam questions after 9th grade. Therefore, the task is appropriate for teaching in lower secondary school. It is evident from the quotation above that the general task, \( \frac{a}{b} + \frac{c}{d} \), is not a part of the mathematical content in lower secondary school.

The pupils are supposed to develop methods of calculation, in this case addition of fractions. This means that choosing and developing methods (technique) depends on the individual pupil even though the methods of course to some extend has to be informal examples more or less similar to more formal methods. Pupils can - and this is often appropriate – choose different methods for different tasks and they do not necessarily have to combine the different methods. The choice of technique depends on the milieu (Winsløw, 2012) often chosen by the teacher. Common Goals 2009 demand that “the work includes mental arithmetic, arithmetic with written notes and use of calculator” (Common Goals 2009, p 26). It is – to some extend – up to the teacher and the individual pupil together to decide e.g. whether the pupil shall use a calculator or written notes to a given task. It is obvious that this choice determines the potential learning outcome.

Formulations on technology in the curriculum are also very open. The thought behind pupils developing methods is that they are supposed to justify and frame explanations to their methods, even though the degree of consistency is fluctuating. The demands for consistency and precision are of course not as high as in scholar mathematics but through a generalization of the technology the pupils will optimize their techniques without necessarily ending out with formulations similar to the scholar
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mathematics but as precise and consistent as possible for the pupil in the
given situation.

Theory is mostly described in informal terms as reasoning and argumentation in Common Goals 2009 – mathematical proofs are only mentioned in a very few cases. It is up to the teacher to decide to what extent the individual elements of the technology are linked, in order to create a common theory based on the individual pupil and the class. Mixed ability classes can achieve different depths in their theoretical way of thinking.

Addition of fractions can also be described and analyzed with the 4T-model from an academic perspective. This includes well-known questions like: What is addition? What is rational numbers? Etc. and definition of algebraic structures including groups and number fields, the compositions + and · and proves of theorems like \((\mathbb{Q}, +, \cdot)\) is a number field. This will not be elaborated further in this paper.

The two praxeologies in the right side of the model (figure 2), knowledge actually taught and knowledge actually learnt, are concerning teaching practice and must be examined and described using the 4T-model by the student teachers in the teaching practice - the former by descriptions of observations of the teacher and the latter by interviews of e.g. pupils, description of pupils based on observations and/or assessments. These two praxeologies are very often based on descriptions from the teaching practice made by the student teachers – this will not be exemplified here.

The descriptions and analysis of addition of rational numbers from the four perspectives (the four kinds of knowledge) in the didactical transposition states a REM to plan, teach and evaluate lessons but also to develop and improve education in general. After describing and analyzing different elements in mathematics teacher education by the 4T-model similar elements or blocks in different kinds of mathematical knowledge can be compared e.g. technology in scholar mathematics can be compared with technology in knowledge actually learnt and techniques in knowledge actually taught can be compared with techniques in knowledge meant to be taught. The comparisons do not always have to go from left to the right in the didactic transposition.
model (figure 2) – observations from teaching practice as e.g. the theory block in knowledge actually learnt can be used to problematize the knowledge meant to be taught in the curriculum.

5. A model of mathematical teacher education

ATD provides a framework for analyzing the theory-practice problem in mathematics teacher education. The two models of Chevallard for didactic transposition process (figure 2) and for praxeology (figure 3) is combined in this section to form a model for analyzing the theory-practice problem in teacher education (figure 4). In my research (a Ph.D. project) the model is intended to be used both descriptively as an analytical tool and normatively as a basis for designing course of lessons in teacher education.

Subsequently, the analysis will be followed by proposals of new ways to organize teaching practice, preparatory education and the theoretical education at the university college to improve the coherence between theory and practice in teacher education.

The model (figure 4) is based on the didactic transposition in schools. As previously mentioned there is a didactic transposition in teacher education as well but the model is constructed to use for student teachers together with teacher educators to describe and analyze the didactic transpositions in schools as this problem area – as I see it – includes all parts of relevant mathematical teacher knowledge.

![Figure 4. Teacher education model](image-url)
The model consists of four columns containing the four kinds of knowledge in the didactic transposition. Each kind of knowledge is described by a mathematical and a didactic praxeology. By collocating the model and teacher education practice three different, pivotal theory-practice problems can be located – occurring in different forms. In the model, the three theory-practice problems in mathematics teacher education are emphasized by red axes – two vertical and one horizontal axis.

The horizontal axis is dividing the practice block and the theory block. This axis stresses the divide between practical, procedural mathematics with emphasis on techniques to carry out tasks and theoretically doing mathematics by combining techniques and concepts, arguing, proving etc. The transcendence of this barrier is a crucial point for mathematical education – the higher level of abstraction in the theoretical block is a necessity but also a very difficult barrier to almost all pupils. Consequently, this axis is a significant problem area for teacher education both with regard to students learning scholar mathematics and pupils learning mathematics at school and the relation between practice and theory block is an appropriate model in both cases.

The two vertical theory-practice axes are dividing, respectively, the scholar mathematics and knowledge meant to be taught and knowledge meant to be taught and knowledge actually taught. The divide in the first axis is treated at the university college. Comparison of scholar mathematics and knowledge meant to be taught is again highly relevant in teacher education to analyze what and why specific content is or is not selected for curriculum. It is pivotal for student teachers to be critical to this selection and to question the decisions in curriculum or textbooks. The arrow at the base of the model pointing “back” from knowledge meant to be taught to scholar mathematics stresses that knowledge meant to be taught can be taken as a starting point for an analysis of a scholar mathematical topic as illustrated in the example in the previous section.

The latter of the vertical axes is dividing the theoretical education taking place at the university college and teaching practice at schools. To combine these two, university colleges teacher education often organize preparatory education as a special forum. Selecting knowledge actually
taught from knowledge meant to be taught is everyday work for teachers and thus obvious content in mathematical teacher education. Again, the arrow is pointing both ways stressing that it is fruitful to move the opposite way and analyze mathematical meant to be taught on the basis of student teachers observations or descriptions of mathematical actually taught or actually learnt in teaching practice.

The two columns to the right are a little different compared to the other kind of knowledge. The relation between knowledge actually taught and knowledge actually learnt cannot offhand be described as a theory-practice problem. As the transposition takes place inside school it is a part of the internal transposition but knowledge actually taught and knowledge actually learnt are closer connected and appears in a more direct interrelationship than the other kinds of knowledge. Student teachers are supposed to react to pupils’ communication and learning e.g. during a dialogue in the classroom and adapt the teaching to the individual pupil or the specific class. Knowledge actually taught and knowledge actually learnt can be theoretically analyzed separately but are in practice intertwined. In the model, the two are therefore not separated but has a common borderline.

A course plan for teacher education can be planed or described based on each of the four praxeologies corresponding to the four columns in the model. The model can be used both descriptively, as shown in the previous section, as a tool to describe e.g. the coherence between the scholar mathematics and the knowledge meant to be taught and normatively, for instance as a tool to relate assessment explicit to knowledge meant to be taught or scholar mathematics to examine if the underlying reason to teach a given content knowledge can be traced in the assessment.

6. Conclusion

A model to correlate theory and practice in teacher education is built on the basis of the anthropologic theory of the didactic. Each of the four kinds of mathematical knowledge: scholar mathematics, knowledge meant to be taught, knowledge actually taught and knowledge actually learnt, can be described and analyzed by the notion of mathematical
praxeologies and didactic praxeologies. By using the same notion in the four different cases student teachers, teachers at university colleges and researchers will be provided a tool to analyze and compare the four praxeologies and thus analyze and create links between three different theory-practice axes located in teacher education. It is a crucial point that the model has arrows in both directions. The study of practice is supposed to be on the basis of new studies in the two columns to the left at the university college for instance by questioning if the contents of knowledge actually taught and the knowledge actually learnt are appropriate in proportion to the knowledge meant to be taught and the scholar mathematics. Course plans in teacher education do not have to take scholar mathematics as starting point – all four columns can be starting points of an analysis or a course plan.

References


