Pathways of change in 1st graders single-digit addition strategies

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Introduction and theoretical framework

Strategy development as described by Siegler's overlapping waves model (1996) is characterised by a continuously changing repertoire of coexisting strategies. This model of strategy change encompass four aspects of strategic competence (Lemaire & Siegler, 1995): 1) repertoire, 2) frequency, 3) efficiency, and 4) adaptivity of strategy choice.

A major goal for primary mathematics education is the efficient and flexible application of various strategies on different types of mathematical tasks (e.g. Verschaffel, Greer, & De Corte, 2007). Student's level of adaptive expertise (Hatano, 2003) is thus important for further development. Teaching aimed at supporting students' development of adaptive and flexible strategy choice in arithmetic problem solving is key to development for students of all achievement levels (Torbeyns, Verschaffel, & Ghesquiere, 2004; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009).

Adaptive strategy choice, i.e. selecting the strategy that most accurately and rapidly produces an adequate answer (Verschaffel et al., 2009), has been described in several studies. The vast majority of research on adaptivity and flexibility has been on strategy choice at a single time point and/or changes in relative frequency of strategies used (e.g. Bailey, Littlefield, & Geary, 2012; Sunde, Sunde, & Sayers, 2019; Torbeyns et al., 2004). However, a poorly understood perspective is how adaptive and flexible strategy choice changes over time and the actual direction of strategy choice on a task level and whether these patterns are the same for boys and girls respectively.

In this paper, I investigate these developmental pathways, more specifically the relative probability by which different strategies (e.g. 'counting all') to solve the same single-digit addition task (e.g. '4+5') are maintained or replaced by other strategies within a timeframe of 5 month in year one. A previous study (Sunde et al., 2019) found strong differences in boys' and girls' strategy use in year one, so therefore this analysis is split on sex. This knowledge has possible implications for research and theory on students' development of adaptive flexibility in arithmetic and direct implications for teaching arithmetic in the early years.

Methods

Strategy use in mental addition for 32 boys and 40 girls (7 years old) was assessed twice in year one with five months' in between (time point 1 in November 2015 and time point 2 in April 2016). In one-to-one assessment interviews, the students were presented with 36 flashcards with an addition problem with numbers 2-9 (Table 4.4). The 36 tasks represented all the possible combinations of the numbers 2 to 9 including tie sums, but only one of each pair of commutative tasks, ensuring an equal amount of tasks with the larger addend first and vice versa.

Strategy use was scored in one of the six categories: a) Error (student gives up or miscalculates), b) Counting all, c) Counting on, d) Direct retrieval, e) Derived facts + (decomposing addends and subsequent use of addition, e.g. 7 + 8 = 7 + 7 + 1), and f) Derived facts − (decomposing addends and subsequent use of subtraction, e.g. 7 + 8 = 8 + 8 − 1).

Since the students solved the same 36 items twice, it was possible to quantify the frequency by which the students solved the same item (e.g. 5 + 8) with the same or different strategies at point 1 and point 2. From, this it was possible to calculate the probabilities that a given strategy (e.g. count all) used at point 1 would be used again on the same item five months later, or replaced by one of the five other strategies. Boys and girls were analysed separately.

Results and implications

In both sexes, 79% of the items solved with direct retrieval at point 1 would also be solved with this strategy at point 2. This indicate that direct retrieval to a large extent are not replaced by other strategies, when first applied to solve a given task. For the remaining 31% of cases, 10% was solved with counting on for both sexes. Girls solved 8% with counting all and 4% with derived fact + or –, whereas boys solved 3% with counting all or error and 9% with derived fact + or –.

If students applied counting on at time point 1, they rarely fell back at using counting all at time point 2 (< 10%). They either used counting on again (boys 51%, girls 69%), or derived fact + and − strategies (boys 42%, girls 21%). Girls tended to change strategies less frequently (more slowly on a time scale) than boys as seen for counting on is even more noticeable for counting all as girls are more than twice as likely as boys to use counting all at time point 2 if they used this strategy at time point 1 (boys 20%, girls 48%).

An interesting pattern appears in the change from using derived fact + at time point 1. At time point 2 girls will primarily use derived fact + (45%) or counting on (32%). This pattern of change is very different for boys, as they will most likely use derived fact + (56%), direct retrieval (20%) or equally likely use derived facts – or counting on (10%). Thus, it appears girls are more prone to 'fall back' to less sophisticated strategies (counting on) from derived fact strategies than boys.

From the differences in likelihood that strategy A replaces strategy B to solve tasks from point 1 to point 2 versus B replaces A, the prevailing developmental pathways of strategies used to solve specific tasks are quantified and ranked for boys and girls. These results will be presented and discussed.

Knowledge about these pathways of change in strategy use might have implications for teaching aimed at supporting adaptive and flexible strategy use. Early arithmetic ability and strategy use has been found to be related to students' general achievement in mathematics (Bailey et al., 2012; Torbeyns et al., 2004; Vanbinst, Ghesquière, & De Smedt, 2014), and Bailey et al. (2012) found that early preference for and skill at using a specific strategy predicts later preference. As strategy repertoire is important for developing adaptive expertise it is important to be aware of students who seem to fix their strategy choice early with preferences for less sophisticated strategies like counting.

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